A QUARTERLY MEASURE OF POTENTIAL OUTPUT IN THE NEW EUROPEAN FISCAL FRAMEWORK

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1. Introduction

The commonly agreed Production Function method developed by the European Commission and by the European Council Output Gap Working Group (OGWG) (D’Auria et al., 2010) to measure potential growth and output gap has gained large relevance, both at national and at the EU level. As regards the European dimension, this methodology has become the reference in Stability and Growth Pact (SGP, as reformed by the so-called Six Pack regulation) to estimate structural deficits and the convergence to Medium Term Objective (MTO). The MTOs are defined in structural terms, meaning that they represent cyclically-adjusted general government budget position, net of one-off and other temporary measures, mostly close to balance. According to the preventive arm of the SGP, countries must attain the MTO or be on an appropriate adjustment path towards it so to ensure sustainable public finances and compliance with the 3% of GDP deficit criterion in all but the most unusual adverse circumstances.

At the national level, the general principle of a balanced State budget over the cycle has been inserted in the Constitutional amendment of article 81 voted by the Italian Parliament in March 2012 and has been expressed in terms of the country specific MTO by the reinforced implementing law (L. no. 243/2012). Likewise, artt. 3 and 7 of Law 243/2012, beyond mentioning that public finances are in equilibrium when at MTO, have defined how to monitor fiscal performance, have

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1 Other approaches to estimate output gap and potential output are the ones developed by the OECD (2010) and IMF (De Masi, 1997) which somehow differ from the Commission methodology. The OECD methodology is based on a production function and quarterly figures, but underlying variables are estimated through simple univariate models. By contrast, IMF uses a set of flexible methodologies (from capacity utilization to real GDP analysis) to estimate potential output and output gaps.

2 The reform process of the EU fiscal surveillance has been completed with two new regulations (the so-called Two Pack) entered into force in May 2013 and regarding new common provisions for monitoring and assessing draft budgetary plans. Overall, the output gap estimation is also needed when evaluating compliance with other fiscal rules, such as the debt rule and the expenditure rule.
introduced a correction mechanism in case of deviation from fiscal target and allowed to deviate from the MTO in exceptional events.

Despite the prominent role reached by the CAB as a tool of fiscal policy, the output gap methodology may be subject to some shortcomings from both theoretical and technical side (Larch and Turrini, 2009). There is also a need to implement timely procedures to assess potential output projections.

First of all, the use of annual data may result in an inappropriate use of available statistical information at higher frequency (quarterly or monthly) that may have some relevance in the derivation of potential growth and output gap in real time.

Secondly, as the out-of-sample extension of potential output components such as Capital, Labour and Total factor productivity, currently carried out to minimize the so-called 'end point bias' of the underlying statistical filter, is performed through simple univariate autoregressive methodologies for a number of variables (such as hours worked, investments and participation rates), it is not possible to take into account over the medium term the cross correlations and linkages among such underlying variables.

Finally, potential output estimates are, typically, carried out assuming the macroeconomic outlook of Commission Spring of Autumn forecasts (or, alternatively, national authorities projections) as exogenous. This choice may result in large revisions of underlying figures due to unavoidable judgmental forecast errors as well as to the huge variability of the out-of-sample projections. Such real time variability may be extremely harmful for policymakers when assessing the achievement of their own MTOs in compliance with European and, in some cases, national constitutional rules.

The relevance of all of these issues is well known and widely recognized especially in technical fora. Recently, we have observed an increasing interest on the methodologies based on mixed frequency models. They are particularly useful to extract the information content from high frequency indicators that are used as proxy for target variables observed at lower frequency and eventually with a time lag. In addition, these models are particularly suited as a time series disaggregation tool, given their multivariate nature and given that the target variable is estimated at a higher frequency.

Relying on these considerations, we propose a new methodology based on the Production Function approach which uses a flexible State Space mixed frequency model (annual plus quarterly) to estimate each factor of production (Labour, Capital and TFP) for determining the level of potential output in real time. In

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3 This approach has been followed by Mariano and Murasawa (2003), Mittnik and Zadrozny (2004), Aruoba et al. (2009), Camacho and Perez Quiros (2009) and Frale et al. (2009). These models can also be used as a multivariate tool for time series disaggregation, as done in Frale et al. (2008), Harvey and Chung (2000), Moauro and Savio (2005).
addition, we also propose a multivariate model using a mixed frequency Kalman filter to extend out of sample in a multivariate framework the pattern of hours worked and participation rates. The advantage of using a mixed frequency model rests on the fact that available and timely information may efficiently be used to provide more reliable real-time estimates of potential output with respect to those obtained through low-frequency annual data. In addition, our proposed model is flexible enough as it could be estimated by imposing external constraints, such as the convergence on annual values such as Commission or national authorities forecasts. Finally, the Kalman filter specification allows to derive a common factor model that may be crucial for extrapolating labour supply variables over longer out-of-sample horizons.

The note is organised as follows. Section 2 presents the mixed frequency model. Section 3 describes the application to Italy showing estimation results and sensitivity analysis. The reliability of our results in real time is also assessed by comparing the variability of potential growth with respect to that obtained by the European Commission through different forecast vintages. Section 4 presents some concluding remarks.

2. The Mixed Frequency Model

Although the methodology currently agreed at the European level for the estimation of the potential output is comprehensive and well established, two possible directions for improvement deserve to be explored: first, the use of quarterly data,\(^4\) and, second, the adoption of a multivariate factor model for estimating potential labour.

The use of disaggregated information allows to exploit timeliest and more updated information as yearly data are released with substantial delay and only once a given year is ended. For instance, yearly data on GDP, let's say for the year 2012, are published only in March 2013, whereas the first information about GDP for the first quarter of 2012 is already available in May 2012. This means that the information contained in partial quarterly figures could be efficiently used for updating the yearly projections, at least, 10 months in advance. This is particularly relevant in periods of high variability of business cycle such as recessions or quick

\(^4\) In the past, the European Commission tried to propose output gap measures based on higher frequency data (i.e. quarterly figures) which, notably, are considered as more suited for estimating business cycles. The adoption of yearly averages of quarterly (or monthly) business survey figures to estimate Total Factor Productivity is an example which goes in this direction. Moreover, the shift to the Bayesian Kalman filter approach to estimate Total Factor Productivity can be considered as a successful attempt to minimize the end point over the medium run.
expansions, when the macroeconomic situation could quickly deteriorate or improve.

Moreover, it is well known in the literature that business cycle features are better captured by high frequency series, quarterly or monthly, which are more sensitive to changes in the business economic activity. By contrast, annual data fail to take into account such underlying variability. This is the reason why, to date the cycle either monthly series (such as the industrial production index) or quarterly data (e.g. GDP) are generally used instead of yearly figures.

As far as the second innovation is concerned, we reckon fundamental to use a multivariate model in order to properly forecast potential labour. It is unambiguously established that participation rate, employment, active population and hours worked are correlated both on the basis of macroeconomic theory and on the basis of the statistical definition of such variables. In our view, using multivariate models guarantees internal consistency among different drivers of the labour market. By contrast, the use of single equations to estimate out of sample labour market dynamics does not guarantee the coherence of single forecasts and thus it can bring to misleading conclusions.

In this respect, the use of mixed frequency models allows to solve, simultaneously, both of the issues identified above that is: using the most recent and updated information to estimate potential output in real time (quarterly) and estimate labour supply relations using a multivariate framework. In addition, the mixed frequency approach and, in particular, the Kalman filter are enough flexible to allow the introduction of some constraints so as to be consistent with predetermined yearly aggregates (such as for example EuroPOP 2010 demographic projection, or EC forecasts).

There are many possibilities for linking a set of indicators available at high frequency to the target variable observed at lower time interval. Among them the mixed frequency factor model proposed in Frale et al. (2008) features an institutional relevance, given that it has been developed by Eurostat for EuroMIND, the Monthly INDicator of the economic activity in the Euro Area. The methodology relies on the idea that a set of time series, available at different frequencies (e.g. quarterly and yearly), can be decomposed into one (or more) common non-stationary component and some idiosyncratic specific to each series. Both the common factor and the idiosyncratic follow autoregressive standard processes as shown by the following representation.

The treatment of the time disaggregation is made following Harvey (1989) and assuming that the disturbances have a Gaussian distribution, the unknown parameters are estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter.
Given the multivariate nature of the model and the mixed frequency constraint, the maximum likelihood estimation can be numerically complex. Therefore, the univariate filter and smoother for multivariate models proposed by Koopman and Durbin (2000) is used as it provides a very flexible and convenient device for handling high dimension and missing values (see Cacciotti, Frale and Teobaldo (2013) for the full model specification).

3. Application for Italy

The methodology presented in the paper has been applied to the Italian case in order to estimate the Output gap, the Potential GDP growth and the relative contributions of labour, capital and total factor productivity. Given the annual data provided by the EC, we use quarterly series by ISTAT or Eurostat so as to disaggregate (to the quarterly frequency) yearly values in sample and to produce quarterly forecast out of sample. This is particularly useful in order to analyse the robustness of the results respect to different projection scenarios.

In section 3.1 we present the results of the disaggregation and forecast of potential GDP and we compare them to the EC’s results (aggregating our quarterly results to yearly values).

Each key input of potential GDP, namely potential labour (LP), capital (K) and (TFP) is estimated in sample at the quarterly frequency and forecasts are produced on different time horizons. The potential GDP is then computed through the classical formula:

\[ \bar{Y} = LP^{0.65} \times K^{0.35} \times SRK. \] (1)

It has to be stressed that an important feature of the model is the fact that it allows not only to match quarterly estimates to be consistent with annual historical data but also to impose out of sample constraints. In fact, the model allows either to exactly replicate the EC forecast, or, alternatively, to constraint only historical data or to impose different constraints on different variables. For example, since the commonly agreed methodology by EC uses the AWG (Aging Working Group) projection to extrapolate the population of working age after the short term forecast horizon, that constraint can be easily included in the model.

In order to show that, we present in Section 3.2 some sensitivity analysis on the stability of the estimates with respect to their revision by applying different input forecasts and between successive EC forecast vintages.

The results allows to appreciate the strengths of the proposed methodology in terms of flexibility and robustness.
3.1. **Estimation results**

This section deals with the detailed presentation of the results of our estimates with respect to the EC forecast exercise of Spring 2012. The main methodological changes are for the Labour and Capital components whereas the TFP is computed with the standard EC model just using a quarterly version of the EC model.

We show how to use timely quarterly data and how to build the multivariate mixed frequency models for Labour and Capital so as to exploit efficiently the cross correlation among data underlying series. We estimate factor loadings along with their standard errors so as to obtain disaggregated quarterly series as resulting from the model. Finally we collect all results and compute the potential output and output gap with the standard official procedure by the EC.

The current methodology applied by the EC for the estimation of potential labour involves several steps. In each of them, a singular component of the total labour supply is estimated through univariate approaches which foresee a mechanical or a simplistic extrapolation of projections out of sample, unless for the NAIRU. The univariate estimates are then plugged into the following equation for the computation of potential Labour:

\[
LP = POPW \times PARTS \times (1 - NAIRU) \times HOURST
\]

where POPW is the active population, PARTS is the smoothed participation rate, and HOURST is the trend of the average of hours worked.

We see a strong limitation in the use of single equations for each component that eventually can produce inconsistent results. Not least, the use of single univariate estimates just plugged into equation (2) open the issue of how to incorporate the statistical errors induced by the single univariate estimates in the final equation, which is completely ignored by the official EC procedure.

To improve the efficiency of the estimates we propose a multivariate dynamic factor model for all labour series, where the different components of the labour supply are jointly estimated and forecast maintaining the coherence among them.

As discussed before we bind yearly series to match EC series and we complement the information with more recent quarterly data. In particular in the application we used annual series of Employment, Unemployment rate, Active Population and Hours worked and quarterly series of Hours worked and Participation rate so as to disaggregate in sample yearly data and to include more recent information.\(^5\)

\(^5\) We tried to include other quarterly series but they come out to be not statistically significant.
The quarterly series of Participation rate and hours worked are those published by ISTAT and the participation rate is consistent with the definition used by the EC and calculated as follow:

\[ PAR = \frac{Empl + Unempl}{POPW} \]  

(3)

where Empl is the total employment, Unempl is the total unemployment and POPW is the total population between 15 and 64 years.

The rationale beyond the use of a multivariate dynamic factor model is to extract a common factor representing the underlying pattern of the labour supply to which the different series are correlated accordingly to a specific factor loading. The results are shown in table 1 where the estimated factor loadings are presented together with their standard errors. The common factor \( D\mu_t \) has been assumed to follow an AR(1) process which is quite standard in the literature.

**Table 1 – Labour market model: Estimated factor loadings with standard errors**

<table>
<thead>
<tr>
<th></th>
<th>Loading</th>
<th>SE</th>
<th>Student-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment</td>
<td>0.09</td>
<td>0.02</td>
<td>3.70</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2.31</td>
<td>0.44</td>
<td>5.21</td>
</tr>
<tr>
<td>DPopulation</td>
<td>0.13</td>
<td>0.02</td>
<td>7.06</td>
</tr>
<tr>
<td>Hours worked</td>
<td>-0.02</td>
<td>0.01</td>
<td>3.58</td>
</tr>
<tr>
<td>Common factor:</td>
<td>(1-0.72 (D\mu_t) = (\eta_t)) (\eta_t \sim N(0,1))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The model produces directly quarterly values of hours worked, while quarterly participation rate is calculated through equation (3). These results are in turn used for extrapolating the series out of sample over the next 6 years. Hence potential levels of both series (hours worked and participation rate) are extracted by applying HP filter. This is only a preliminary attempt and other filters such as Kalman or Band-pass can be applied to improve the quality of results. The NAIRU quarterly series is obtained by appropriately changing the parameters of the GAP program by the EC. For this preliminary exercise NAIRU quarterly series has been obtained by converting the annual frequencies (data from Commission) by a quadratic-match average method, but it could be envisaged a quarterly version of the standard bivariate model for the NAIRU. To compare our results to those obtained by the EC, we aggregate the potential labour series by averaging the quarterly information.

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6 The population has been included in the model in first difference so as to match the cyclical characteristic of the other series which generally are more dynamic.

7 However, we are currently working on a different specification of the multivariate model for labour supply allowing to forecast also the series of wage growth. On the basis of this specifications, the quarterly NAIRU can be projected out of sample also for the period \((t+3) - (t+5)\).
over a yearly frequency. Figure 1 shows Italian potential labour over the period 1981-2016 as obtained by the EC compared with our estimates.

**Figure 1 – Potential Labour**

As expected the quarterly method produces slightly more volatile results given the higher frequency of the data. Moreover, the inclusion of updated quarterly values for the year 2012 (up to first quarter of 2012) allows to better capture the slowdown due to the recent economic recession.

As far as the estimation of Capital is concerned, we rely on the EC model at the yearly level and we disaggregate the series at the quarterly frequency by using a multivariate model similar to that used for the Labour supply. In particular, we use quarterly data on Investments published by ISTAT to disaggregate the yearly series of Capital taking into account also yearly potential output as estimated in a first run of the procedure as to mimic the practice in the EC’s approach. We would like to emphasize that Investments and Potential Output are not the focus of the model but only instruments to disaggregate yearly Capital at the quarterly frequency. For sake of brevity we omit the full results which are available under request.

Once labour supply and capital stock are estimated, Solow residual and the corresponding estimate of the Total factor Productivity at quarterly frequencies can be computed. In order to do that, we use a quarterly version of the program GAP
by the EC, where prior distributions at the quarterly frequency have been derived accordingly.

The Solow residual is calculated until the end of the short term forecast horizon by using quarterly forecast of GDP obtained by applying a multivariate model similar to that of Labour consistently with yearly EC's projections for the years 2012 and 2013. The quarterly capacity index used as a proxy for the unobserved level of capacity utilization is the usual CUBS of the EC's procedure calculated at quarterly frequency by transforming indicators coming from the Business and Tendency Surveys published by the EC from a monthly to a quarterly frequency. Figure 2 shows our results compared with those of the EC.

**Figure 2 – Quarterly trend total factor productivity**

As before the quarterly frequency allows to better capture the cyclical swings and thus to produce a more dynamic TFP.

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8 See for more details the web site http://ec.europa.eu/economy\_finance/db\_indicators/surveys/index\_en.htm.
Combining the results obtained in previous steps we compute the quarterly potential output by using the Cobb-Douglas production function (1). Results are shown in figure 3 in levels and growth rates along with the estimated value of the potential GDP by EC in Spring 2012.

Our results shows a lower potential output growth after the crisis and consequently a smaller output gap.

**Figure 3 – Potential output and output gap: MEF vs EC Spring 2012 estimates**

3.2. *Analysis of revisions and sensitivity to forecasts*

In this section, we present an insight on the stability of our estimates with respect to the revision of the variables and consistently with the updating of available data. Moreover, the impact of changes in short term forecasts of input series on long term growth prospects is also assessed. Figure 4 presents the estimates of potential output resulting by applying different input forecasts.

In particular, whereas the constraint to historical data is always maintained, we assume different ways to link the model to yearly EC forecast data for the period 2012-2016.
More in details:

- **Case 1**: The model is constrained to the 2012 Spring forecasts over the period 2012-2013 unless for Active Population that is linked to EUROPOP 2010 projection up to 2018. In such scenario, the only difference with respect to the commonly agreed Production Function methodology is due to the use of quarterly data in the multivariate model for Labour Supply and in both Capital and TFP components.

- **Case 2**: The model keeps the link to historical figures but it is set to freely produce forecasts for all the underlying series with the only exception of Active Population which is still constrained to EUROPOP projections up to 2018.

- **Case 3**: also the constraint on the active population is relaxed and the model produces the forecast for all variables from 2012 up to 2016.

**Figure 4 – Sensitivity Analysis: Impact of forecasts**

The inspection of figure 4 shows the relevance of Active population in defining the long term growth.

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**Note:**
- Case 1: Historical data up to 2011 + Spring forecast 2012-2013 + Active Population up to 2018, the difference is due only to the new methodology.
- Case 2: Historical data up to 2011 + forecasts by the model, unless Active Population constraint up to 2018.
- Case 3: Historical data up to 2011 + forecasts by the model for all variables including Active Population.
In fact estimates by Case 1 and Case 2 are quite similar and the only relevant change occurs once the constraint to the Active population is relaxed.

Moreover, the differences in sample with EC estimates are due to the use of quarterly values which allows to capture cyclical swings and thus it produces a more dynamic output growth.

A second experiment of sensitivity analysis is made in terms of revision of estimates between successive vintages of EC Forecasts.

**Figure 5 – Sensitivity Analysis: Revisions of estimates**

Figure 5 presents the estimates of Potential Output (in growth rates) in the last three consecutive vintages of forecast: Spring 2011, Autumn 2011 and Spring 2012. The same vintages specification is presented for our method. It is clearly visible that the proposed new methodology appears to be less influenced by revision of data especially on an historical basis. Whereas the EC methodology produces substantial revisions which extends backward until 2000, our model shows stable results for statistical historical figures. In addition, as our model is bounded to the results of EC forecasts, the revisions in the outer part of the sample mainly represent the forecast error underlying the projection exercise in each vintage. Moreover it has to be highlighted that another source of revision with
respect to 2012 EC Spring forecast is represented by the introduction in our estimates of the latest figures on the first quarter of 2012. This produces a drop in potential growth for 2012 which is not reflected in the latest EC forecast.

Finally, we propose an ex-post evaluation to provide an overview of the benefit of using quarterly information which is available in advance respect to yearly figures. In fact, although the EC updates the macroeconomic forecast twice per year this is not enough to incorporate new information released during the year. Figure 6 compares the estimate of the Output gap made in December 2012 with both the official EC procedure and the MEF quarterly method with the same realized in February 2013 that exploits all the information about 2012 published in the National Accounts.

**Figure 6** – Comparison of EC and MEF results with ex-post estimated values of Output Gap

It is quite visible how the quarterly procedure outperforms the standard approach in producing values closer to those made with full information and this is due probably to the ability of the quarterly method to exploit partial information on the year 2012. We are aware that this is only a single example but given the complexity of the exercise we leave for a future analysis a proper assessment of the real time performance of our results.
4. Conclusion

This methodology, still in progress, presents a new way of estimating on a quarterly basis the single components of potential output (Labour, Capital and TFP).

For Capital and Labour a multivariate State Space model in mixed frequency has been adopted. Though computationally more demanding, this specification, through the use of quarterly and annual observations, is able to reproduce and timely update - more often than under the current OGWG framework - both the historical, the real time and the projected series of the European Commission forecasts.

As it is mostly based on higher frequency observations, our methodology allows also to capture business cycle features and the variability of economic fluctuations in a more efficient fashion than what would result by using annual data.

Besides all this, one of the most important innovations of our methodology is represented by the use of a multivariate Kalman Filter for projecting out of sample all the components of labour supply (including wage growth) and Capital Stock. The choice of a multivariate framework for projecting jointly out of sample (over the years \((t+3)-(t+5)\)) hours worked, participation rates, unemployment rates (and eventually wage growth) allows to exploit the underlying macroeconomic relations existing, respectively, among the components of Labour supply and Capital stock and to provide a sound alternative to the simple univariate procedures in use so far.

According to our estimates, results for Italy appear as more robust and stable than those obtained with the current methodology at least as far as past historical revisions of underlying figures of different forecast vintages are concerned. As shown by Cacciotti and Pradelli (2009), revisions of past values of unobserved variables are potentially large and may surely affect the results in real time. In such a context, the relative stability in potential output growth estimates for past observations is a desirable feature especially for its use in the current fiscal framework for determining the medium term average growth of the expenditure benchmarks and assess the attainment of the Medium Term Objective (MTO) as prescribed by the new constitutional amendment introduced in the Italian legislation.

In our opinion, this model offers some appealing features. In particular, it allows to assess on a quarterly basis the reliability of real time of output gaps and potential growth based on underlying annual macroeconomic projections. Such a property appears as being essential in a fiscal framework, such as the one introduced in Italy in 2012, where the compliance to the MTO is crucial to assess whether particular correction mechanisms should be triggered or not on the basis of real time variables and a specific macroeconomic medium term outlook. In
addition, a quarterly framework based on mixed frequency variables allows to assess the revision in output gaps and structural deficits due to the updating of macroeconomic variables, providing the policy makers with an efficient tool to measure possible slippages from the MTO well in advance and giving to them the possibility to reshape fiscal policies in case of need.

Acknowledgement

This paper represents the authors personal opinions and does not reflect the view of the Italian Department of Treasury. We are grateful to an anonymous referee for his comments. We also thank people working at the Treasury-Dir I-Unit II for their support in the elaboration of the results. Routines on the mixed frequency factor model are coded in Ox 3.3 by Doornik (2001) and are based on the programs realized by Tommaso Proietti for the Eurostat project on EuroMIND: the Monthly Indicator of Economic Activity in the Euro Area.

5. References


SUMMARY

This paper presents a new mixed frequency methodology to estimate output gaps and potential output on a quarterly basis. The methodology strongly relies on the production function method commonly agreed at the European level (D'Auria et. al., 2010) but it significantly improves it allowing to assess the impact of real time forecast for GDP and other underlying variables. This feature of the model is particularly welcome in the current Italian budgetary framework which has foreseen the introduction of the principle of a budget balance in structural terms in the Constitution. By allowing to measure output gap with a quarterly span on the basis of recent developments indicators, the methodology provides interesting hints on the cyclical position of the economy in real time to be used for deriving cyclically-adjusted fiscal aggregates.

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